Exercises from Chapter 6

Example  (page 175: 18) A 5.0 \times 10^4\text{-kg space probe is traveling at a speed of 11000 m/s through deep space. Retrorockets are fired along the line of motion to reduce the probe’s speed. The retrorockets generate a force of 4.0 \times 10^5 N over a distance of 2500 km. What is the final speed of the probe?

The work done by the rockets is

\[ W = -(4.0 \times 10^5)(2500000) = -1.0 \times 10^{12} \text{ J} \]

This is also the change in kinetic energy of the probe.

\[ \Delta KE = -1.0 \times 10^{12} \text{ J} \]

The initial kinetic energy of the probe is

\[ KE_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(5.0 \times 10^4)(11000^2) = 3.025 \times 10^{12} \text{ J} \]

The final kinetic energy of the probe is

\[ KE_f = KE_0 + \Delta KE = 3.025 \times 10^{12} - 1.0 \times 10^{12} = 2.025 \times 10^{12} \text{ J} \]

\[ \frac{1}{2}mv_f^2 = 2.025 \times 10^{12} \text{ J} \]

\[ v_f = \sqrt{\frac{2(2.025 \times 10^{12})}{m}} = \sqrt{\frac{2(2.025 \times 10^{12})}{5.0 \times 10^4}} = 9000 \text{ m/s} \]

Example  (page 177: 46) A basketball player makes a jump shot. The 0.600-kg ball is released at a height of 2.00 m above the floor with a speed of 7.20 m/s. The ball goes through the net 3.10 m above the floor at a speed of 4.20 m/s. What is the work done on the ball by air resistance, a nonconservative force?

The initial mechanical energy of the basketball is
\[ E_0 = KE_0 + PE_0 \]
\[ = \frac{1}{2}mv_0^2 + mgh_0 \]
\[ = \frac{1}{2}(0.600)(7.20^2) + (0.600)(9.80)(2.00) \]
\[ = 27.312 \text{ J} \]

The final mechanical energy of the basketball is

\[ E_f = KE_f + PE_f \]
\[ = \frac{1}{2}mv_f^2 + mgh_f \]
\[ = \frac{1}{2}(0.600)(4.20^2) + (0.600)(9.80)(3.10) \]
\[ = 23.52 \text{ J} \]

The work done by the nonconservative forces is equal to the change in energy.

\[ W_{nc} = \Delta E = 27.312 - 23.52 = 3.79 \text{ J} \]

**Example** (page 177: 44) The drawing shows a version of the loop-the-loop trick for a small car. If the car is given an initial speed of 4.0 m/s, what is the largest value that the radius \( r \) can have if the car is to remain in contact with the circular track at all times?

![Diagram of a small car on a loop-the-loop track]

The initial mechanical energy of the car is

\[ E_0 = KE_0 + PE_0 = \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}m16.0 = 8.0m \]

The mechanical energy at point \( A \) is

\[ E_A = KE_A + PE_A = \frac{1}{2}mv_A^2 + mg2r \]

By conservation of mechanical energy, \( E_0 = E_A \).
We re-write this equation to get

\[ mv_A^2 = m(16 - 4gr) \]

The centripetal force at point \( A \) is

\[ F_c = \frac{mv_A^2}{r} = \frac{m(16 - 4gr)}{r} \]

In the largest possible loop, the centripetal force at \( A \) is exactly equal to the gravitational force.

\[ \frac{m(16 - 4gr)}{r} = mg \]

From this equation we get \( r = 0.33 \text{ m} \).

**Example** (page 176: 40) The drawing shows a skateboarder moving at 5.4 m/s along a horizontal section of a track that is slanted upward by 48° above the horizontal at its end, which is 0.40 m above the ground. When she leaves the track, she follows the characteristic path of projectile motion. Ignoring friction and air resistance, find the maximum height \( H \) to which she rises above the end of the track.

The initial mechanical energy is

\[ E_0 = KE_0 + PE_0 = \frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}m5.4^2 + 0 = 14.58m \]

The mechanical energy at the end of the ramp is

\[ E_f = \frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_f^2 + m(9.8)(0.40) = \frac{1}{2}mv_f^2 + 3.92m \]
Since mechanical energy is conserved, $E_0 = E_f$.

$$14.58m = \frac{1}{2}mv_f^2 + 3.92m$$

So the speed at the end of the ramp is $v_f = 4.62$ m/s.

After the skateboarder leaves the ramp she is in free fall. To describe her motion, we can use the kinematics equations

$$v_y = v_{0y} + a_y t$$
$$y = v_{0y} t + \frac{1}{2} a_y t^2$$

The initial velocity $v_0$ as the skateboarder leaves the ramp is equal to the velocity we just found $4.62$ m/s. The $y$ component of this velocity is

$$v_{0y} = 4.62 \sin 48^\circ = 3.43$$

Filling in the equation $v_y = v_{0y} + a_y t$ for the highest point we get

$$0 = 3.43 - 9.8t$$

We solve for the time to get $t = 0.35$ s. Next, from the equation $y = v_{0y} t + \frac{1}{2} a_y t^2$ we get

$$y = (3.43)(0.35) + \frac{1}{2}(-9.8)(0.35^2) = 0.60 \text{ m}$$

**Impulse and Momentum**

The simple kinematics equation $x = v_0 t + \frac{1}{2} a t^2$ gives complete information about the motion of a particle provided the acceleration is constant. If, however, the acceleration is not constant, then this equation may become much more complex. Equations that were easy to solve may be turned into equations that are difficult to solve or even impossible to write down. Thus, in our study of motion, we are also on a search for ways to simplify. This was part of our motivation for studying energy. Constants are almost always simpler than variables. Knowing that energy, in many cases, is constant, reduced the complexity of our work.

In this section we consider momentum which is another conserved quantity. Momentum, defined as $mv$, is built from the same basic parts as kinetic energy $\frac{1}{2}mv^2$. There are, however, situations in which kinetic energy is not conserved but momentum is.
**Definition** The momentum of an object of mass $m$ and velocity $v$ is

$$p = mv$$

If the force applied to an object is not constant, then it is convenient to work with the average force:

$$\bar{F} = m\bar{a} = m\left(\frac{v_f - v_0}{\Delta t}\right)$$

If we multiply both sides of the equation by $\Delta t$, we get

$$\bar{F}\Delta t = m(v_f - v_0) = mv_f - mv_0$$

$$\bar{F}\Delta t = p_f - p_0 = \Delta p$$

So the change in momentum is equal to the average force times the change in time. This result leads to another definition.

**Definition** The impulse of a force $F$ acting over a time $\Delta t$ is

$$J = \bar{F}\Delta t$$

Using this definition, we can now state the impulse-momentum theorem:

**impulse = change in momentum**

**Example** (page 199: 1) Two identical automobiles have the same speed, one traveling east and one traveling west. Do these cars have the same momentum? Explain.

*The two momenta have the same magnitude but different directions. Therefore they are not equal.*

**Example** (page 199: 2) In Times Square in New York City, people celebrate on New Year’s Eve. Some just stand around, but many move about randomly. Consider a system consisting of all of these people. Approximately, what is the total linear momentum of this system at any given instant? Justify your answer.

*On average, the velocity of the people is approximately zero. Therefore the momentum is approximately zero.*

**Example** (page 199: 3) Two objects have the same momentum. Do the velocities of these objects necessarily have (a) the same directions and (b) the same magnitudes? Give your reasoning in each case.
The direction of the momentum is the same as the direction of the velocity. Therefore the two momenta have the same direction. The velocities do not need to have the same magnitude. For example, object one may have velocity \( 2 \text{ m/s} \) and mass \( 2 \text{ kg} \). If object two has velocity \( 1 \text{ m/s} \) and mass \( 4 \text{ kg} \), then the two objects have the same momentum but different velocities.

Example (page 199: 4) (a) Can a single object have kinetic energy but no momentum? (b) Can a system of two or more objects have a total kinetic energy that is not zero but a total momentum that is zero? Account for your answers.

Since kinetic energy is \( \frac{1}{2}mv^2 \) and momentum is \( mv \), an object that has kinetic energy must also have momentum. It is possible, however, for a system of objects to have nonzero kinetic energy but zero momentum. Consider a system of two objects of equal mass \( m \) with velocities \( 3 \text{ m/s} \) and \( -3 \text{ m/s} \). The total kinetic energy of the system is \( KE = \frac{1}{2}m9 + \frac{1}{2}m9 = 9m \). The momentum of the system is \( m3 - m3 = 0 \).

Example (page 199: 5) An airplane is flying horizontally with a constant momentum during a time interval \( \Delta t \). (a) With the aid of Equation 7.4, decide whether a net impulse is acting on the plane during this time interval. (b) In the horizontal direction, both the thrust generated by the engines and air resistance act on the plane. What does the answer in part (a) imply about the impulse of the thrust compared to the impulse of the resistive force?

Since there is no change in momentum, there is also no impulse. The impulse of the thrust and the impulse of the air resistance must be of equal magnitude and in opposite directions.

Example (page 199: 6) 1. You have a choice. You may get hit head-on either by an adult moving slowly on a bicycle or by a child that is moving twice as fast on a bicycle. The mass of the child and bicycle is one-half that of the adult and bicycle. Considering only the issues of mass and velocity, which collision do you prefer? Or doesn’t it matter? Account for your answer.

Both bicycles have the same momentum, but the child has more kinetic energy. Therefore, it is likely better to collide with the adult.