Potential Energy

How can you increase the kinetic energy of a baseball? By throwing it. Since $KE = \frac{1}{2}mv^2$, an increase in speed produces an increase in energy. A less direct method is to lift the baseball as high as you can and then drop it. As the ball falls its speed increases and so does its kinetic energy. Assuming the ball is dropped from rest at a height $h$, the final velocity of the ball is given by

$$v^2 = v_0^2 + 2ay = 0 + 2ay = 2gh$$

So the final kinetic energy of the ball (just before it hits ground) is

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh \quad (1)$$

In the previous section, we learned that the total work done on an object is equal to its change in kinetic energy. So suppose you are working very hard to lift a baseball to a height $h$. On the way up you are supplying a force to offset the effect of gravity. Since the lift force and the displacement are in the same direction, the work you are performing is positive. But when you are done lifting, after all that positive work, the baseball is just sitting in your hand with kinetic energy of zero. What happened? How did you fail to change the kinetic energy of the ball? The answer is that while you were doing positive work, gravity was doing an equal amount of negative work so the total work on the ball is zero.

Let’s calculate how much energy was “lost.” The amount of work done is $W = Fs$. Since the applied force is equal to the weight of the baseball $mg$ and the displacement is the height $h$, we have

$$W = Fs = mgh$$

If we had done this much work on the baseball without gravity, the kinetic energy of the ball would also be $mgh$. With gravity, the kinetic energy has been reduced to zero.

There’s no need to worry about this lost energy. Just release the ball and the energy will return. According to equation (1), the kinetic energy of the ball just before it hits ground is $mgh$ which is exactly the amount of “lost” energy.

Suppose you inherit one million dollars from a great uncle you never met but who decided to include you in his will anyway. Upon receiving the money you spent half of it on a new house. Does buying the house make you $500000 poorer? Not really, because you could sell the house and get the money back.

Our baseball is in a similar situation. When it is above your head, resting in your hand, its kinetic energy is zero. But for accounting purposes, we should also count the energy it
could have if you were to release it. Therefore, we invent a new category of energy known as potential energy. We define this potential energy to be

\[ \text{PE} = mgh \]

which is exactly the amount of kinetic energy we could get by dropping the ball.

**Conservative Forces**

Suppose a manufacturing company wants to ship its finished goods from Los Angeles to Seattle. Would it make any sense for the company to ship the product first to New York and then to Seattle rather than going to Seattle directly? Probably not. The longer trip would require more work. More work means more energy and more energy means higher cost.

So experience tells us that the amount of work done depends on the path taken. Gravity, however, is different. The work gravity does depends only on where the object starts and where it stops but not on the route it takes. Imagine you have just been promoted to vice president of a large manufacturing company in Los Angeles (for reducing shipping costs of course). Your large, well-furnished office is on the top floor of one of the many high-rise skyscrapers. Each day at 5:00 as you ride the elevator down to ground level, gravity performs positive work on you equal to \( mgh \). If the elevators are broken one day and you are forced to take the stairs, then your path changes. Instead going straight down, you now move along a spiral. What is the work that gravity did on the day you took the stairs? Still \( mgh \). The work done by gravity only depends on where you start and where you stop.

We will now divide force into two kinds. Forces that do the same amount of work regardless of the path that joins the starting point and ending point are known as conservative forces. All other forces are called nonconservative.

A ready example of a nonconservative force is friction. Gravity is our usual example of a conservative force.

If we label the work done by conservative forces \( W_c \) and the work done by nonconservative forces \( W_{nc} \), then the total work done is \( W = W_c + W_{nc} \). By the work-energy theorem, the total work is equal to the change in kinetic energy. Thus,

\[ W_c + W_{nc} = \Delta \text{KE} \quad (2) \]

For gravity, the work done as an object moves upward a height \( h \) is \( W_c = -mgh \) while its final potential energy is \( mgh \). This suggests the equation \( W_c = -\Delta \text{PE} \) which in fact turns out to be true. Thus, equation (2) can now be re-written.

\[ -\Delta \text{PE} + W_{nc} = \Delta \text{KE} \]
which is equivalent to

\[ W_{nc} = \Delta KE + \Delta PE \]

**Example**  (page 176: 26) A 0.60-kg basketball is dropped out of a window that is 6.1 m above the ground. The ball is caught by a person whose hands are 1.5 m above the ground.

(a) How much work is done on the ball by its weight? What is the gravitational potential energy of the basketball, relative to the ground, when it is (b) released and (c) caught?

(d) How is the change \((PE_k - PE_0)\) in the ball’s gravitational potential energy related to the work done by its weight?

(a) \[ W = Fs = mg\Delta h = (0.60)(-9.8)(1.5 - 6.1) = 27 \text{ J} \]

(b) \[ PE_0 = mgh = (0.60)(9.8)(6.1) = 35.9 \text{ J} \]

(c) \[ PE_f = mgh = (0.60)(9.8)(1.5) = 8.8 \text{ J} \]

(d) The work done by gravity is opposite the change in potential energy.

**Conservation of Mechanical Energy**

The mechanical energy of an object is defined to be the sum of its kinetic and potential energy.

\[ E = KE + PE \]

From the equation \( W_{nc} = \Delta KE + \Delta PE \), we get

\[ W_{nc} = \Delta E \]

In words, the change in mechanical energy equals the work done by nonconservative forces. If \( W_{nc} = 0 \), then \( 0 = \Delta E \). So the mechanical energy does not change. This is known as the conservation of mechanical energy.

**If the net work done by all nonconservative forces is zero then mechanical energy \( E \) is constant.**
Example  (page 176: 32) A gymnast is swinging on a high bar. The distance between his waist and the bar is 1.1 m, as the drawing shows. At the top of the swing his speed is momentarily zero. Ignoring friction and treating the gymnast as if all of his mass is located at his waist, find his speed at the bottom of the swing.

Since we are ignoring friction, the only nonconservative force is the centripetal force applied by the gymnast’s arms. Since this force is perpendicular to the displacement, it does no work. Therefore, mechanical energy is conserved.

At the top of the swing, the energy is \( E = KE + PE = 0 + mg2.2 \). At the bottom of the swing, the energy is \( E = KE + PE = \frac{1}{2}mv^2 + 0 \). Since energy is conserved, these two expressions must be equal.

\[
mg2.2 = \frac{1}{2}mv^2
\]

\[
v = \sqrt{2g2.2} = \boxed{6.6 \text{ m/s}}
\]