Review for Test 1

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An object is thrown upward at an angle $\theta$ above the ground, eventually returning to earth. (a) Is there any place along the trajectory where the velocity and acceleration are perpendicular? If so, where? (b) Is there any place where the velocity and acceleration are parallel? If so, where. In each case, explain.

The velocity of the object is always changing and is always in the direction of the motion. The acceleration, however, is a constant $-9.80 \text{ m/s}^2$ downward. The velocity and acceleration are perpendicular when the object is at its highest point. They will never be parallel unless $\theta = 90^\circ$.

![Diagram of object trajectory with acceleration vector](attachment:object_trajectory.png)

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A tennis ball is hit upward into the air and moves along an arc. Neglecting air resistance, where along the arc is the speed of the ball (a) a minimum and (b) a maximum? Justify your answers.

The acceleration of the ball is $-9.80 \text{ m/s}^2$. When the ball is moving up, it is slowing down. When the ball is moving down, it is speeding up. The speed of the ball will be maximum at its two lowest points and minimum at its highest point.

8. A rifle, at a height $H$ above the ground, fires a bullet parallel to the ground. At the same instant and at the same height, a second bullet is dropped from rest. In the absence of air resistance, which bullet strikes the ground first? Explain.

Notice that the two bullets have exactly the same $y$ equations:

$$y = -H, \quad v_{0y} = 0, \quad a_y = -9.80 \text{ m/s}^2$$

Therefore, the two bullets strike the ground at the same time.

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A leopard springs upward at a $45^\circ$ angle and then falls back to the ground. Does the leopard, at any point on its trajectory, ever have a speed that is one-half its initial value? Give your reasoning.
If $v_0$ is the initial speed of the leopard, then $v_{0x} = v_0 \cos 45^\circ = 0.707v_0$. Since $v_{0x}$ is constant throughout the trajectory, the speed of the leopard will never fall below this value. Therefore, the minimum speed of the leopard is about 71% of its initial speed.

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The altitude of a hang glider is increasing at a rate 6.80 m/s. At the same time, the shadow of the glider moves along the ground at a speed of 15.5 m/s when the sun is directly overhead. Find the magnitude of the glider’s velocity.

The magnitude of the velocity is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{15.5^2 + 6.80^2} = 16.9 \text{ m/s}$.

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You are traveling in a convertible with the top down. The car is moving at a constant velocity of 25 m/s, due east along flat ground. You throw a tomato straight upward at a speed of 11 m/s. How far has the car moved when you get a chance to catch the tomato?

For the tomato we have $v_{0y} = 11 \text{ m/s}$ and $a_y = -9.80 \text{ m/s}^2$. Starting with the equation $y = v_{0y}t + \frac{1}{2}a_yt^2$, we set $y = 0$ and solve for $t$.

\[
0 = 11t + \frac{1}{2}(-9.8)t^2 = 11t - 4.9t^2
\]

\[
0 = t(11 - 4.9t)
\]

\[
t = 0 \quad \text{or} \quad t = 11/4.9 = 2.2 \text{ s}
\]

The tomato is thrown at time $t = 0$ s and it is caught at $t = 2.2$ s. During this time the car has moved $(25)(2.2) = 55 \text{ m}$.

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In the javelin throw at a track-and-field event, the javelin is launched at a speed of 29 m/s at an angle of 36° above the horizontal. As the javelin travels upward, its velocity points above the horizontal at an angle that decreases as time passes. How much time is required for the angle to be reduced from 36° at launch to 18°?
The initial velocity is $v_{0x} = 29 \cos 36^\circ = 23.5 \text{ m/s}$ and $v_{0y} = 29 \sin 36^\circ = 17.0 \text{ m/s}$. The final velocity (when the angle is $18^\circ$) is $v_x = v_{0x} = 23.5 \text{ m/s}$ and $v_y = v_{0y} + ayt = 17.0 - 9.8t$. Since the angle is $18^\circ$, we have

$$\frac{17.0 - 9.8t}{23.5} = \tan 18^\circ = 0.325$$

$$17.0 - 9.8t = (23.5)(0.325)$$

$$t = 0.96 \text{ s}$$

Stones are thrown horizontally with the same velocity from the tops of two different buildings. One stone lands twice as far from the base of the building from which it was thrown as does the other stone. Find the ratio of the height of the taller building to the height of the shorter building.

Let $y_a$ be the height of the shorter building and $y_b$ the height of the taller. The stone thrown from the taller building must have taken twice the amount of time to fall as the stone thrown from the shorter building. Since the initial $y$ velocity for both stones is zero, we have

$$y_a = \frac{1}{2} ay^2 \text{ and } y_b = \frac{1}{2} ay(2t)^2$$

The ratio of the heights is

$$\frac{y_b}{y_a} = \frac{1}{2} ay(2t)^2 = 4$$

The drawing shows an exaggerated view of a rifle that has been “sighted in” for a 91.4-meter target. If the muzzle speed of the bullet is $v_0 = 427 \text{ m/s}$, what are the two possible angles $\theta_1$ and $\theta_2$ between the rifle barrel and the horizontal such that the bullet will hit the target? One of these angles is so large that it is never used in target shooting. (Hint: The following trigonometric identity may be useful: $2 \sin \theta \cos \theta = \sin 2 \theta$.)

The initial velocity is $v_{0x} = 427 \cos \theta$ and $v_{0y} = 427 \sin \theta$. The position of the bullet is $x = (427 \cos \theta)t$ and $y = (427 \sin \theta)t - 4.90t^2$. To find when the bullet hits the target, we set $x = 91.4 \text{ m}$.
\[ 91.4 = (427 \cos \theta) t \]

\[ t = \frac{91.4}{427 \cos \theta} = \frac{0.2141}{\cos \theta} \]

We next substitute this value of \( t \) into the \( y \) equation.

\[ y = (427 \sin \theta) \left( \frac{0.2141}{\cos \theta} \right) - 4.90 \left( \frac{0.2141}{\cos \theta} \right)^2 \]

We then set \( y \) to zero.

\[ 0 = (427 \sin \theta) \left( \frac{0.2141}{\cos \theta} \right) - 4.90 \left( \frac{0.2141}{\cos \theta} \right)^2 \]

Multiply both sides of the equation by \((\cos \theta)^2\).

\[ 0 = 427 \sin \theta \cos \theta (0.2141) - (4.90)(0.2141)^2 \]

Divide both sides by 0.2141.

\[ 0 = 427 \sin \theta \cos \theta - (4.90)(0.2141) \]

\[ \sin \theta \cos \theta = \frac{4.90(0.2141)}{427} \]

Multiply both sides by 2.

\[ 2 \sin \theta \cos \theta = \frac{2(4.90)(0.2141)}{427} = 0.004914 \]

Use the hint.

\[ \sin 2\theta = 0.004914 \]

\[ 2\theta = \sin^{-1} 0.004914 = 0.2815 \]

\[ \theta = \frac{0.2815}{2} = \frac{0.141^\circ}{2} \]
To find the other value of $\theta$, we solve the equation

$$2\theta = 180 - 0.2815 = 179.7185$$

$$\theta = \frac{1}{2}(179.7185) = 89.9^\circ$$