Vector Addition

**Example**  Suppose that \( \mathbf{v} \) is a vector with magnitude 7 and that the angle of \( \mathbf{v} \) with the positive \( x \)-axis is 15°.

![Diagram of vector \( \mathbf{v} \) with components \( v_x \) and \( v_y \).]

The components of \( \mathbf{v} \) are given by the formulas

\[
\begin{align*}
v_x &= 7 \cos 15^\circ = 6.761 \\
v_y &= 7 \sin 15^\circ = 1.812
\end{align*}
\]

The vector \( \mathbf{v} \) is the sum of its components: \( \mathbf{v} = v_x + v_y = (v_x, v_y) = (6.761, 1.812) \).

Suppose we have another vector \( \mathbf{w} \) with magnitude 3 and angle 90°.

![Diagram showing vectors \( \mathbf{v} \) and \( \mathbf{w} \).]

From the picture, we see that the components of \( \mathbf{w} \) are \( w_x = 0 \) and \( w_y = 3 \). Thus, \( \mathbf{w} = (0, 3) \).

Let \( \mathbf{R} \) be the sum \( \mathbf{v} + \mathbf{w} \). To represent the sum graphically, we line up the initial point of \( \mathbf{w} \) with the terminal point of \( \mathbf{v} \):
Algebraically, \( \mathbf{R} \) is the sum:

\[
\mathbf{R} = \mathbf{v} + \mathbf{w} = (6.761, 1.812) + (0, 3) = (6.761, 4.812)
\]

The magnitude of \( \mathbf{R} \) is given by the Pythagorean theorem: 

\[
|\mathbf{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{6.761^2 + 4.812^2} = 8.30.
\]

To find the angle of \( \mathbf{R} \), we use the equation \( \tan \theta = \frac{R_y}{R_x} \). Applying the inverse tangent we have

\[
\theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{4.812}{6.761} = 35.4^\circ
\]

In this example we started with two vectors given in terms of their magnitude and direction. We then proceeded to find their components. Having the components, we could add them to find the resultant vector \( \mathbf{R} \). Finally, we used the equations \( |\mathbf{R}| = \sqrt{R_x^2 + R_y^2} \) and \( \theta = \tan^{-1}(R_y/R_x) \) to find the magnitude and direction of the resultant vector.

**Example** A person is walking on the deck of a ship with a velocity of \( \mathbf{w} = (-2, 1) \) m/s relative to the ship. The ship is moving relative to the shore with a velocity of \( \mathbf{v} = (3, 7) \) m/s. What is the velocity of the person relative to the shore?

The velocity of the person relative to the shore is the sum \( \mathbf{R} = \mathbf{v} + \mathbf{w} = (3, 7) + (-2, 1) = (1, 8) \) m/s.

**Example** A bicyclist is heading due east. A wind of 5.0 m/s is blowing in the direction 35° north of west. What component of the wind is against the cyclist?

The component of the wind against the cyclist is \( 5.0 \cos 35^\circ = 4.1 \) m/s.

**Example** A basketball player runs 7.0 m in a straight line, turns 45° and runs another 7.0 m. Finally, the player runs 7.0 m in the same direction that he originally ran. What is the total displacement of the basketball player?

We will call the three displacement vectors \( \mathbf{A}, \mathbf{B}, \) and \( \mathbf{C} \).
The components of the vectors are

\[ \mathbf{A} = (7.0, 0) \text{ m} \]

\[ \mathbf{B} = (7.0 \cos 45^\circ, 7.0 \sin 45^\circ) = (4.95, 4.95) \text{ m} \]

\[ \mathbf{C} = (7.0, 0) \text{ m} \]

The resultant vector is

\[ \mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = (18.95, 4.95) \text{ m} \]

The magnitude of the displacement is \( \sqrt{18.95^2 + 4.95^2} = 20 \text{ m} \). The angle of the displacement is \( \tan^{-1}(4.95/18.95) = 15^\circ \) away from the direction of the final displacement.