1. The mass of the parasitic wasp *Caraphractus cintus* can be as small as \(5 \times 10^{-6} \text{ kg}\). What is this mass in (a) grams (g), (b) milligrams (mg), and (c) micrograms (µg)?

(a) \(5 \times 10^{-6} \text{ kg} = 5 \times 10^{-6} \text{ kg} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} = 5 \times 10^{-3} \text{ g}\)

(b) \(5 \times 10^{-3} \text{ g} = 5 \times 10^{-3} \text{ g} \cdot \frac{1000 \text{ mg}}{1 \text{ g}} = 5 \text{ mg}\)

(c) \(5 \text{ mg} = 5 \text{ mg} \cdot \frac{1000 \text{ µg}}{1 \text{ mg}} = 5 \times 10^{3} \text{ µg}\)

3. How many seconds are there in (a) one hour and thirty-five minutes and (b) one day?

(a) \(1 \text{ hour and 35 minutes} = (60 + 35) \text{ min} = 95 \text{ min} = 95 \text{ min} \cdot \frac{60 \text{ s}}{1 \text{ min}} = 5700 \text{ s}\)

(b) \(1 \text{ day} = 1 \text{ day} \cdot \frac{24 \text{ h}}{1 \text{ day}} \cdot \frac{3600 \text{ s}}{1 \text{ h}} = 86400 \text{ s}\)

5. The largest diamond ever found had a size of 3106 carats. One carat is equivalent to a mass of 0.200 g. Use the fact that 1 kg (1000 g) has a weight of 2.205 lb under certain conditions, and determine the weight of this diamond in pounds.

\[
3106 \text{ carats} = 3106 \text{ carats} \cdot \frac{200 \text{ g}}{1 \text{ carat}} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} \cdot \frac{2.205 \text{ lb}}{1 \text{ kg}} = 1370 \text{ lb}
\]

7. The following are dimensions of various physical parameters that will be discussed later on in the test. Here [L], [T], and [M] denote, respectively, dimensions of length, time, and mass.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Dimension</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance ((x))</td>
<td>[L]</td>
<td>Acceleration ((a))</td>
</tr>
<tr>
<td>Time ((t))</td>
<td>[T]</td>
<td>Force ((F))</td>
</tr>
<tr>
<td>Mass ((m))</td>
<td>[M]</td>
<td>Energy ((E))</td>
</tr>
<tr>
<td>Speed ((v))</td>
<td>([L]/[T])</td>
<td></td>
</tr>
</tbody>
</table>
Which of the following equations are dimensionally correct?

(a) \( F = ma \)  \hspace{1cm} (b) \( x = \frac{1}{2}at^3 \)  \hspace{1cm} (c) \( E = \frac{1}{2}mv \)  \hspace{1cm} (d) \( E = max \)  \hspace{1cm} (e) \( v = \sqrt{Fx/m} \)

(a), (d), and (e) are dimensionally correct.

8. The variables \( x \), \( v \), and \( a \) have the dimensions of \([L]\), \([L]/[T]\), and \([L]/[T]^2\), respectively. These variables are related by an equation that has the form \( v^n = 2ax \), where \( n \) is an integer constant (1, 2, 3, etc.) without dimensions. What must be the value of \( n \), so that both sides of the equation have the same dimensions? Explain your reasoning.

<table>
<thead>
<tr>
<th>variables</th>
<th>dimensions</th>
<th>simplified dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v^n )</td>
<td>( \left( \frac{[L]}{[T]} \right)^n )</td>
<td>( \frac{[L]^n}{[T]^n} )</td>
</tr>
<tr>
<td>( 2ax )</td>
<td>( \frac{[L]}{[T]^2} )</td>
<td>( \frac{[L]^2}{[T]^2} )</td>
</tr>
</tbody>
</table>

By comparing the entries in the last column, we see that \( n = 2 \).

9. The depth of the ocean is sometimes measured in fathoms (1 fathom = 6 feet). Distance on the surface of the ocean is sometimes measured in nautical miles (1 nautical mile = 6076 feet). The water beneath a surface rectangle 1.20 nautical miles by 2.60 nautical miles has a depth of 16.0 fathoms. Find the volume of water (in cubic meters) beneath this rectangle.

\[
\text{volume} = (4815 \text{ m})(2222 \text{ m})(29.26 \text{ m}) = 3.13 \times 10^8 \text{ m}^3
\]

11. You are driving into St. Louis, Missouri, and in the distance you see the famous Gateway-to-the-West arch. This monument rises to a height of 192 m. You estimate your line of sight with the top of the arch to be 2.0° above the horizontal. Approximately how far (in kilometers) are you from the base of the arch?
13. A highway is to be built between two towns, one of which lies 35.0 km south and 72.0 km west of the other. What is the shortest length of highway that can be built between the two towns, and at what angle would this highway be directed with respect to due west?

\[ d = \sqrt{35.0^2 + 72.0^2} = 80.1 \text{ km} \]
\[ \theta = \tan^{-1} \frac{72.0}{35.0} = 64.1^\circ \]
\[ 90^\circ - 64.1^\circ = 25.9^\circ \text{ south of west} \]

15. The silhouette of a Christmas tree is an isosceles triangle. The angle at the top of the triangle is 30.0°, and the base measures 2.00 m across. How tall is the tree?

\[ \tan 15^\circ = \frac{1.00}{h} \]
\[ h = \frac{1.00}{\tan 15^\circ} = 3.73 \text{ m} \]

19. What is the value of each of the angles of a triangle whose sides are 95, 150, and 190 cm in length? (*Hint: Consider using the law of cosines.*)
Law of cosines: $150^2 = 95^2 + 190^2 - 2 \cdot 190 \cdot 95 \cos \alpha$

$$
\cos \alpha = \frac{150^2 - 95^2 - 190^2}{-2 \cdot 190 \cdot 95}
$$

$$
\alpha = \cos^{-1} \left( \frac{150^2 - 95^2 - 190^2}{-2 \cdot 190 \cdot 95} \right) = 51.2^\circ
$$

$$
\beta = \cos^{-1} \left( \frac{190^2 - 95^2 - 150^2}{-2 \cdot 150 \cdot 95} \right) = 99.2^\circ
$$

$$
\gamma = \cos^{-1} \left( \frac{95^2 - 150^2 - 190^2}{-2 \cdot 190 \cdot 150} \right) = 29.6^\circ
$$

21. A force vector $\mathbf{F}_1$ points due east and has a magnitude of 200 newtons. A second force $\mathbf{F}_2$ is added to $\mathbf{F}_1$. The resultant of the two vectors has a magnitude of 400 newtons and points along the east/west line. Find the magnitude and direction of $\mathbf{F}_2$. Note that there are two answers.

The vector $\mathbf{F}_1$ in component form is $(200, 0)$. The vector $\mathbf{F}_2$ in component form is $(f_1, f_2)$. The resultant in component form is $(\pm 400, 0)$. Thus, we have the equation

$$(200, 0) + (f_1, f_2) = (\pm 400, 0)$$

This gives us the equations

$$
200 + f_1 = \pm 400
$$

$$
f_2 = 0
$$

Solving the equations, we get $f_2 = 0$ and $f_1 = 200$ or $f_1 = -600$. If $f_1 = 200$ then $\mathbf{F}_2$ has magnitude $200 \text{ N}$ and direction east. If $f_1 = -600$ then $\mathbf{F}_2$ has magnitude $600 \text{ N}$ and direction west.

22. (a) Two workers are trying to move a heavy crate. One pushes on the crate with a force $\mathbf{A}$, which has a magnitude of 445 newtons and is directed due west. The other
pushes with a force \( \mathbf{B} \), which has a magnitude of 325 newtons and is directed due north. What are the magnitude and direction of the resultant force \( \mathbf{A} + \mathbf{B} \) applied to the crate? (b) Suppose that the second worker applies a force \( -\mathbf{B} \) instead of \( \mathbf{B} \). What then are the magnitude and direction of the resultant force \( \mathbf{A} - \mathbf{B} \) applied to the crate? In both cases express the direction relative to due west.

(a) The force vectors are shown below.

The magnitude of the resultant is \( \sqrt{445^2 + 325^2} = 551 \text{ N} \).

The angle of the resultant is

\[
\theta = \tan^{-1} \left( \frac{325}{445} \right) = 36.1^\circ \text{ north of west}
\]

(b) The magnitude of the resultant is still \( 551 \text{ N} \) but the direction is now \( 36.1^\circ \text{ south of west} \).

24. The drawing shows a triple jump on a checkerboard, starting at the center of square A and ending on the center of square B. Each side of a square measures 4.0 cm. What is the magnitude of the displacement of the colored checker during the triple jump?

The checker moves 8.0 cm to the right and 24.0 cm up. The displacement is
31. The speed of an object and the direction in which it moves constitute a vector quantity known as the velocity. An ostrich is running at a speed of 17.0 m/s in a direction of 68.0° north of west. What is the magnitude of the ostrich’s velocity component that is directed (a) due north and (b) due west?

(a) \(17.0 \sin 68.0° = 15.8 \text{ m/s}\)

(b) \(17.0 \cos 68.0° = 6.37 \text{ m/s}\)

41. A golfer, putting on a green, requires three strokes to “hole the ball.” During the first putt, the ball rolls 5.0 m due east. For the second putt, the ball travels 2.1 m at an angle of 20.0° north of east. The third putt is 0.50 m due north. What displacement (magnitude and direction relative to due east) would have been needed to “hole the ball” on the very first putt?

In component form, the displacements are \((5.0, 0), (2.1 \cos 20°, 2.1 \sin 20°),\) and \((0, .50)\). The sum of the three vectors is

\((5.0, 0) + (2.1 \cos 20°, 2.1 \sin 20°) + (0, .50) = (6.973, 1.218)\)

The magnitude is \(\sqrt{6.973^2 + 1.218^2} = 7.08 \text{ m}\). The angle is \(\tan^{-1}(1.218/6.973) = 9.91° \text{ north of east}\).

43. Find the resultant of the three displacement vectors in the drawing by means of the component method. The magnitudes of the vectors are \(A = 5.00 \text{ m}, B = 5.00 \text{ m},\) and \(C = 4.00 \text{ m}\).

Finding the components:

\(A = (-5.00 \cos 20°, 5.00 \sin 20°) = (-4.698, 1.710)\)
\( \mathbf{B} = (5.00 \cos 60^\circ, 5.00 \sin 60^\circ) = (2.500, 4.330) \)

\( \mathbf{C} = (0, -4.000) \)

The sum of the three vectors is \((-4.698, 1.710) + (2.500, 4.330) + (0, -4.000) = (-2.198, 2.040)\).

\[ \text{The magnitude is } \sqrt{(-2.198)^2 + 2.040^2} = 3.00 \text{ m}. \]

The angle is \( \theta = \tan^{-1}(2.040/2.198) = 42.9^\circ \text{ above the negative x-axis} \).