Review Problems from Chapter 4

Example (page 116: 9) Two forces $\mathbf{F}_A$ and $\mathbf{F}_B$ are applied to an object whose mass is 8.0 kg. The larger force is $\mathbf{F}_A$. When both forces point due east, the object's acceleration has a magnitude of $0.50\,\mathrm{m/s^2}$. However, when $\mathbf{F}_A$ points due east and $\mathbf{F}_B$ points due west, the acceleration is $0.40\,\mathrm{m/s^2}$, due east. Find (a) the magnitude of $\mathbf{F}_A$ and (b) the magnitude of $\mathbf{F}_B$.

Newton's second law states that the sum of the forces equals mass times acceleration.

$$\sum \mathbf{F} = ma$$

This equation can be written as

$$\frac{\sum \mathbf{F}}{m} = a$$

When the two forces are in the same direction we have

$$\frac{F_A + F_B}{8.0} = 0.50$$

which reduces to $F_A + F_B = 4.0$. When the forces are in opposite directions we have

$$\frac{F_A - F_B}{8.0} = 0.40$$

which reduces to $F_A - F_B = 3.2$. We now have a system of equations.

$$F_A + F_B = 4.0$$
$$F_A - F_B = 3.2$$

Adding the equations gives $2F_A = 7.2$ and thus, $F_A = \boxed{3.6\,\mathrm{N}}$. Subtracting the equations gives $2F_B = 0.80$ and thus, $F_B = \boxed{0.40\,\mathrm{N}}$.

Example (page 117: 16) At a time when mining asteroids has become feasible, astronauts have connected a line between their 3500-kg space tug and a 6200-kg asteroid. Using their ship's engine, they pull on the asteroid with a force of 490 N. Initially the tug and the asteroid are at rest, 450 m apart. How much time does it take for the ship and the asteroid to meet?

Acceleration of asteroid: $a = \frac{F}{m} = \frac{490}{6200} = 0.07903\,\mathrm{m/s^2}$

Acceleration of tug: $a = \frac{F}{m} = \frac{490}{3500} = 0.14\,\text{m/s}^2$

The relative acceleration between the asteroid and the tug is $0.07903 + 0.14 = 0.21903\,\text{m/s}^2$.

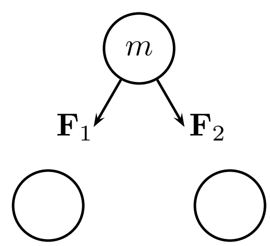
Using the kinematics equation $x = v_0 t + \frac{1}{2}at^2$, we get

$$450 = \frac{1}{2}(0.21903)t^2$$

Solving for t:

$$t = \boxed{64.1\,\text{s}}$$

Example (page 117: 28) Three uniform spheres are located at the corners of an equilateral triangle. Each side of the triangle has a length of 1.20 m. Two of the spheres have a mass of 2.80 kg each. The third sphere (mass unknown) is released from rest. Considering only the gravitational forces that the spheres exert on each other, what is the magnitude of the initial acceleration of the third sphere?



The force between the unknown mass m and one of the other masses is

$$F = \frac{Gm_1 m}{r^2} = \frac{(6.673 \times 10^{-11})(2.80)m}{1.20^2} = 1.298 \times 10^{-10} m$$

The components of the forces are

$\mathbf{F}_1 = (-F \sin 30°, -F \cos 30°) = (-6.49 \times 10^{-11} m, -1.124 \times 10^{-10} m)$

$\mathbf{F}_2 = (F \sin 30°, -F \cos 30°) = (-6.49 \times 10^{-11} m, -1.124 \times 10^{-10} m)$

The sum of these two forces is

$$\mathbf{F}_1 + \mathbf{F}_2 = (0, -2.248 \times 10^{-10} m)$$

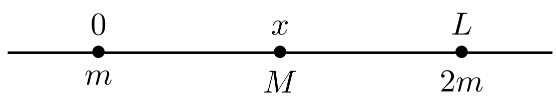
The magnitude is

$$|\mathbf{F}_1 + \mathbf{F}_2| = 2.248 \times 10^{-10} m$$

The acceleration of the third sphere is

$$a = \frac{|\mathbf{F}_1 + \mathbf{F}_2|}{m} = \boxed{2.248 \times 10^{-10}\ \mathrm{m/s^2}}$$

Example (page 117: 33) Two particles are located on the x axis. Particle 1 has a mass m and is at the origin. Particle 2 has a mass $2m$ and is at $x = +L$. A third particle is placed between particles 1 and 2. Where on the x axis should the third particle be located so that the magnitude of the gravitational force on *both* particle 1 and particle 2 doubles? Express your answer in terms of L.



The gravitational force between particles 1 and 2 is

$$F = \frac{G2m^2}{L^2}$$

The gravitational force between particle 1 and particle 3 is

$$F_1 = \frac{GmM}{x^2}$$

Since the force on particle 1 doubles, we must have $F_1 = F$:

$$\frac{GmM}{x^2} = \frac{G2m^2}{L^2} \tag{1}$$

The gravitational force between particle 2 and particle 1 is

$$F_2 = \frac{G2mM}{(L-x)^2}$$

Since the force on particle 2 doubles, we must have $F_2 = F$:

$$\frac{G2mM}{(L-x)^2} = \frac{G2m^2}{L^2} \tag{2}$$

From equations (1) and (2), we get

$$\frac{GmM}{x^2} = \frac{G2mM}{(L-x)^2}$$

which reduces to

$$\frac{1}{x^2} = \frac{2}{(L-x)^2}$$

Cross multiplication gives us the equation $x^2 + 2xL - L^2 = 0$. From the quadratic formula we find

$$x = \frac{-2L \pm \sqrt{4L^2 + 4L^2}}{2} = -L \pm \sqrt{2}L$$

We choose the + solution since x is between 0 and L. Thus,

$$x = (\sqrt{2} - 1)L = \boxed{0.414L}$$

Example (page 118: 45) While moving in, a new homeowner is pushing a box across the floor at a constant velocity. The coefficient of kinetic friction between the box and the floor is 0.41. The pushing force is directed downward at an angle θ below the horizontal. When θ is greater than a certain value, it is not possible to move the box, no matter how large the pushing force is. Find that value of θ.

The components of the pushing force are

$$\mathbf{F} = (F\cos\theta, -F\sin\theta)$$

The normal force on the box is $F_N = mg + F\sin\theta$. The frictional force is $f_k = \mu_k F_N = (0.41)(mg + F\sin\theta)$. We are at the maximum value of θ when the frictional force is equal to the x component of the pushing force.

$$(0.41)(mg + F\sin\theta) = F\cos\theta$$

The box is easier to push when its weight is small. As a limiting case, take the weight to be zero.

$$0.41F\sin\theta = F\cos\theta$$

This equation may be written as

$$\tan\theta = \frac{1}{0.41}$$

So $\theta = \tan^{-1}\dfrac{1}{0.41} = \boxed{67.7^\circ}$.

Example (page 119: 57) A person is trying to judge whether a picture (mass = 1.10 kg) is properly positioned by temporarily pressing it against a wall. The pressing force is perpendicular to the wall. The coefficient of static friction between the picture and the wall is 0.660. What is the minimum amount of pressing force that must be used?

We need the frictional force to equal the weight of the picture.

$$f_k = \mu_k F_N = mg$$

$$(0.660)F_N = (1.10)(9.80)$$

Using this equation we find $F_N = \boxed{16.3\,\text{N}}$. This is the minimum push force required assuming there is no friction between the person's hand and the picture.

Example (page 120: 80) A girl is sledding down a slope that is inclined at 30.0° with respect to the horizontal. A moderate wind is aiding the motion by providing a steady force of 105 N that is parallel to the motion of the sled. The combined mass of the girl and sled is 65.0 kg, and the coefficient of kinetic friction between the runners of the sled and the snow is 0.150. How much time is required for the sled to travel down a 175-m slope, starting from rest?

The weight of the sled is $w = (65.0)(9.80) = 637\,\text{N}$. The component of the weight parallel to the motion is $w_\parallel = 637\sin 30^\circ = 318.4\,\text{N}$. The component of the weight perpendicular to the motion is $w_\perp = 637\cos 30^\circ = 551.7\,\text{N}$. The frictional force is

$$f_k = \mu_k F_N = \mu_k w_\perp = (0.150)(551.7) = 82.76\,\text{N}$$

The sum of all the forces is

$$\sum F = w_\parallel + \text{wind} - f_k = 318.5 + 105 - 82.76 = 340.7\,\text{N}$$

The acceleration is

$$a = \frac{\sum F}{m} = \frac{340.7}{65.0} = 5.242\,\text{m/s}^2$$

Using the kinematics equation $x = \frac{1}{2}at^2$ we find

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(175)}{5.242}} = \boxed{8.17\,\mathrm{s}}$$