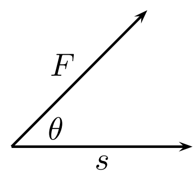
Work

In physics, work is defined to be force times displacement. For example, if you lift a 100-N box from the floor to a shelf 1.5 m above, then the amount of work you have done is $(100)(1.5) = 150$. This definition should seem reasonable since if you double either the weight of the box or the height of the shelf, then the amount of work done is doubled. Suppose you get a new job in which your only task is to move boxes from the floor to a shelf. In such a job, it might make sense to be paid by the newton-meter. That is, the amount of work you have done is measured in terms of how many newtons you have moved and how far you have moved those newtons.

To get a precise definition of work, suppose that a force of magnitude F acts on an object that is displaced a distance s as shown below.



In the picture, the part of the force F in the direction of the displacement is $F\cos\theta$. Therefore we define work to be

$$W = (F\cos\theta)s$$

If F is measured in newtons and s is measured in meters, then W is measured in newton-meters. As an abreviation, one newton-meter is also called a joule which is further abreviated as J.

Example A flag is tied to a flagpole. The wind applies a force of 10 N on the flag. What is the work done by the wind on the flag?

Since the displacement of the flag is zero, the work done by the wind is also zero.

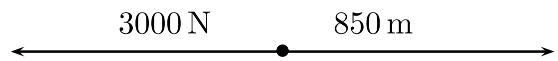
Example The wind applies a force of 10 N on a flag as you carry the flag 3 m directly into the wind. What is the work done by the wind on the flag?

The work done is

$$W = (F\cos\theta)s = (10)(3) = \boxed{30\,\text{J}}$$

Example (page 174: 1) The brakes of a truck cause it to slow down by applying a retarding force of 3.0×10^3 N to the truck over a distance of 850 m. What is the work done by the this force on the truck? Is the work positive or negative? Why?

Since the 3000 N is a retarding force, it is the opposite direction of the displacement.



The angle between the force and the displacement is 180°. Therefore the work done by the brakes is

$$W = (3000 \cos 180°)(850) = \boxed{-2.55 \times 10^6\ \mathrm{J}}$$

The work is negative because $\cos 180° = -1$.

Example (page 174: 4) A 75.0-kg man is riding an escalator in a shopping mall. The escalator moves the man at a constant velocity from ground level to the floor above, a vertical height of 4.60 m. What is the work done on the man by (a) the gravitational force and (b) the escalator?

In computing work, we only use the component of the force that is in the direction of the displacement. In this example, it is easier to instead use the component of the displacement that is in the direction of the force. Thus, $F = (75.0)(9.80) = 735\ \mathrm{N}$ and $s = 4.60\ \mathrm{m}$. Since the gravitational force and the displacement are in opposite directions the work done by gravity on the man is

(a) $$W = -(735)(4.60) = \boxed{-3380\ \mathrm{J}}$$

The work done by the escalator on the man is

(b) $$W = (735)(4.60) = \boxed{3380\ \mathrm{J}}$$

Example (page 175: 8) A 55-kg box is being pushed a distance of 7.0 m across the floor by a force **P** whose magnitude is 150 N. The force **P** is parallel to the displacement of the box. The coefficient of kinetic friction is 0.25. Determine the work done on the box by each of the, four forces that act on the box. Be sure to include the proper plus or minus sign for the work done by each force.

The normal force is $F_N = mg = (55)(9.8) = 539\ \mathrm{N}$. The frictional force is $f_k = \mu_k F_N = (0.25)(539) = 135\ \mathrm{N}$.

The work done by the push force **P** is

$$W = (F \cos \theta)s = (150 \cos 0°)(7.0) = (150)(7.0) = \boxed{1100\ \mathrm{J}}$$

The work done by friction is

$$W = (F\cos\theta)s = (135\cos 180°)(7.0) = (-135)(7.0) = \boxed{-945\,\mathrm{J}}$$

Both the weight and the normal force are perpendicular to the displacement. Therefore the work done by these forces is zero.

Example (page 175: 10) A 1.00×10^2-kg crate is being pushed across a horizontal floor by a force **P** that makes an angle of 30.0° below the horizontal. The coefficient of kinetic friction is 0.200. What should be the magnitude of **P**, so that the net work done by it and the kinetic frictional force is zero?

The push force **P** has two components. There is the component of **P** parallel to the floor $P_\parallel = P\cos 30° = 0.8660P$ which pushes the box forward and there is the perpendicular component $P_\perp = P\sin\theta = 0.5P$ which pushes the box against the floor.

The normal force is the weight of the box plus $P_\perp$.

$$F_N = mg + P_\perp = (100)(9.80) + 0.5P = 980 + 0.5P$$

The frictional force is

$$f_k = \mu_k F_N = (0.200)(980 + 0.5P) = 196 + 0.1P$$

If the net work is zero, then the magnitude of **P** must be equal to f_k.

$$P = f_k$$

$$P = 196 + 0.1P$$

Solving this equation, we get $P = \boxed{218\,\mathrm{N}}$.

Kinetic Energy

Very loosely, we will define energy as the ability to do work. Since there are many sources of work, we will have to consider different kinds of energy. The first that we will consider is called kinetic energy. It is the energy an object has due to its motion.

From experience, it should be clear that a fast moving baseball has more energy than a slow moving baseball. You can feel the difference when you are the catcher. But mass also makes a difference. A baseball of 1.0 kg should have more energy than a baseball of 0.75 kg. Once again, you can feel the difference when you are catching the ball. The definition of kinetic energy accounts for both mass and speed. The definition is

$$\text{kinetic energy} = \text{KE} = \frac{1}{2}mv^2$$

It is clear from this definition that increasing the mass or the speed of an object will increase its kinetic energy. It is not as clear why v is squared and why the factor of $\frac{1}{2}$ is included. The answer is that we would like to establish a connection between kinetic energy and work.

Suppose that $\mathbf{F}$ is the net force acting on an object and that s is the distance the object is displaced in the direction of $\mathbf{F}$. First, from the equation $v_f^2 = v_0^2 + 2as$, we get

$$as = \frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 \tag{1}$$

The total work done on the object is $W = Fs$. Since $F = ma$, we have $W = mas$. Combining this with equation (1), we get

$$W = mas = m\left(\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2\right) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$
$$W = \text{KE}_f - \text{KE}_0$$

In words, this says that the total work done on an object is equal to its change in kinetic energy.

Example (page 175: 12) A fighter jet is launched from an aircraft carrier with the aid of its own engines and a steam-powered catapult. The thrust of its engines is 2.3×10^5 N. In being launched from rest it moves through a distance of 87 m and has a kinetic energy of 4.5×10^7 J at lift-off. What is the work done on the jet by the catapult?

The work done on the jet is equal to its change in kinetic energy.

$$W = \Delta\text{KE}$$

The work done by the engines is

$$W_e = Fs = (2.3 \times 10^5)(87) = 2.00 \times 10^7 \text{ J}$$

The work W_c done by the catapult is unknown. The change in kinetic energy is 4.5×10^7 J. Putting these parts together, we get

$$W_e + W_c = \Delta\text{KE}$$
$$2.00 \times 10^7 + W_c = 4.5 \times 10^7$$

We solve this equation to get $W_c = \boxed{2.5 \times 10^7\,\text{J}}$.

Example (page 175: 14) Since total work done is equal to change in kinetic energy, we have

$$\begin{aligned} W &= \Delta\text{KE} \\ Fs &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \end{aligned}$$

Since $v_0 = 0$, we get

$$Fs = \frac{1}{2}mv_f^2$$

We solve this equation and obtain

$$v_f = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2(65)(0.90)}{0.075}} = \boxed{39\,\text{m/s}}$$

Example (page 175: 19) A sled is being pulled across a horizontal patch of snow. Friction is negligible. The pulling force points in the same direction as the sled's displacement, which is along the $+x$ axis. As a result, the kinetic energy of the sled increases by 38%. By what percentage would the sled's kinetic energy have increased if this force had pointed 62° above the $+x$ axis?

The work done is equal to the change in kinetic energy. In the first part of the problem, this gives us

$$W_1 = Fs = \Delta\text{KE} = 0.38\text{KE}_0$$

In the second part of the problem when the force is applied at an angle of 62°, the change in kinetic energy is

$$\Delta\text{KE} = W_2 = (F\cos 62^\circ)s = Fs\cos 62^\circ = 0.38\Delta\text{KE}_0\cos 62^\circ = 0.18\Delta\text{KE}_0$$

Thus, there is an 18% increase in kinetic energy.

Example (page 175: 23) An extreme skier, starting from rest, coasts down a mountain that makes an angle of 25.0° with the horizontal. The coefficient of kinetic friction between her skis and the snow is 0.200. She coasts for a distance of 10.4 m before coming to the edge of a cliff. Without slowing down, she skis off the cliff and lands downhill at a point whose vertical distance is 3.50 m below the edge. How fast is she going just before she lands?

While on the slope, the normal force is $F_N = mg\cos 25°$. So the frictional force is $f_k = \mu_k F_N = (0.200)(mg\cos 25°)$. The sum of all forces acting parallel to the slope is

$$F_{\parallel} = mg\sin 25° - f_k = mg\sin 25° - (0.200)(mg\cos 25°)$$

The work done on the skier is

$$W = Fs = F_{\parallel}10.4$$

Since the skier started from rest, the change in kinetic energy is equal to the final kinetic energy $\frac{1}{2}mv^2$. Since work done equals change in kinetic energy, we have

$$F_{\parallel}10.4 = \frac{1}{2}mv^2$$

$$(mg\sin 25° - (0.200)(mg\cos 25°))(10.4) = \frac{1}{2}mv^2$$

Solving this equation for v, we get $v = 7.014\,\text{m/s}$. The vertical component of this velocity is $v_y = -7.014\sin 25° = -2.964\,\text{m/s}$.

In the next part of the problem, as the skier is flying through the air, we have an initial y velocity of $v_{0y} = -2.964\,\text{m/s}$. The acceleration is $a_y = -9.80\,\text{m/s}^2$ and the final y position is $y = -3.50\,\text{m}$. From the equation $v_y^2 = v_{0y}^2 + 2a_y y$, we get

$$v_y = \sqrt{v_{0y}^2 + 2a_y y} = \sqrt{2.964^2 + 2(-9.80)(-3.50)} = \boxed{8.80\,\text{m/s}}$$