Exercises from Chapter 5

Example (page 143: 5) Computer-controlled display screens provide drivers in the Indianapolis 500 with a variety of information about how their cars are performing. For instance, as a car is going through a turn, a speed of 221 mi/h (98.8 m/s) and a centripetal acceleration of $3.00g$ (three times the acceleration due to gravity) are displayed. Determine the radius of the turn (in meters).

The acceleration of the race car is $a = 3.00g = (3.00)(9.80) = 29.4\,\mathrm{m/s^2}$. From the equation $a = v^2/r$, we solve for r.

$$r = \frac{v^2}{a} = \frac{98.8^2}{29.4} = \boxed{332\,\mathrm{m}}$$

Example (page 143: 8) Each of the space shuttle's main engines is fed liquid hydrogen by a high-pressure pump. Turbine blades inside the pump rotate at 617 rev/s. A point on one of the blades traces out a circle with a radius of 0.020 m as the blade rotates. (a) What is the magnitude of the centripetal acceleration that the blade must sustain at this point? (b) Express this acceleration as a multiple of $g = 9.80\,\mathrm{m/s^2}$.

The distance traveled in one revolution is $2\pi r = 2\pi 0.020 = 0.126\,\mathrm{m}$. Therefore, $617\,\mathrm{rev}/scorrespondstoas$

The centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{0.126^2}{0.020} = \boxed{0.79\,\mathrm{m/s^2}}$$

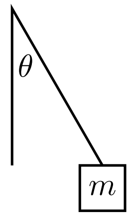
The ratio of this acceleration to g is

$$\frac{0.79}{g} = \frac{0.79}{9.8} = 0.081$$

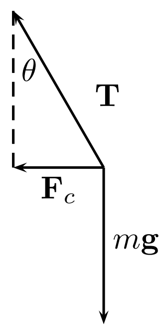
Thus, $a = \boxed{0.081g}$.

Example (page 144: 18) A block is hung by a string from the inside roof of a van. When the van goes straight ahead at a speed of 28 m/s, the block hangs vertically down. But when the van maintains this same speed around an unbanked curve (radius = 150 m), the block swings toward the outside of the curve. Then the string makes an angle θ with the vertical. Find θ.

The picture below shows the block and the string.



The tension in the string, the centripetal force, and gravity (mg) are directed as shown below.



Since the mass is not accelerating up or down, the y component of the tension must be equal to the weight of the mass.

$$T_y = T\cos\theta = mg \tag{1}$$

The x component of the tension supplies the centripetal force.

$$T_x = T\sin\theta = F_c$$

Since $F_c = mv^2 r$, we have

$$T\sin\theta = \frac{mv^2}{r} \tag{2}$$

Dividing equation (2) by equation (1) gives us

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mv^2/r}{mg}$$

This simplifies to

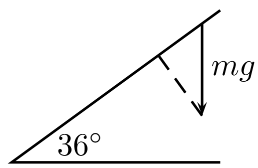
$$\tan\theta = \frac{v^2}{rg}$$

and θ is given by

$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{28^2}{(150)(9.8)} = \boxed{28°}$$

Example (page 144: 26) The drawing shows a baggage carousel at an airport. Your suitcase has not slid all the way down the slope and is going around at a constant speed on a circle ($r = 11.0\,\mathrm{m}$) as the carousel turns. The coefficient of static friction between the suitcase and the carousel is 0.760, and the angle θ in the drawing is 36.0°. How much time is required for your suitcase to go around once?

The weight of the suitcase is shown below. The normal force is parallel to the dotted line.



The maximum frictional force is

$$f_s^{\mathrm{MAX}} = \mu_s F_N = 0.760 F_N$$

In component form, the frictional force is

$$\mathbf{f}_s^{\mathrm{MAX}} = (0.760F_N\cos 36.0°, 0.760F_N\sin 36.0°) = (0.6149F_N, 0.4467F_N)$$

The normal force is 54.0° above the negative x axis. In component form the normal force is

$$\mathbf{F}_N = (-F_N\cos 54.0°, F_N\sin 54.0°) = (-0.5878F_N, 0.8090F_N)$$

The components of the gravitational force are

$$m\mathbf{g} = (0, -mg) = (0, -9.80m)$$

The sum of all forces acting on the suitcase is

$$\begin{aligned}&\mathbf{f}_s^{\mathrm{MAX}} + \mathbf{F}_N + m\mathbf{g}\\ &= (0.6149F_N, 0.4467F_N) + (-0.5878F_N, 0.8090F_N) + (0, -9.80m)\\ &= (0.0271F_N, 1.256F_N - 9.80m)\end{aligned}$$

Since the suitcase is not accelerating up or down, we must have $1.256F_N - 9.80m = 0$. Solving this equation, we get $F_N = 7.803m$.

The x component of the total force, $0.0271F_N$, must be equal to the centripetal force mv^2/r. Thus,

$$0.0271F_N = \frac{mv^2}{r}$$

Replacing F_N with $7.803m$, we get

$$(0.0271)(7.803)m = \frac{mv^2}{r}$$

From this equation, we solve for v.

$$v = \sqrt{(0.0271)(7.803)r} = \sqrt{(0.0271)(7.803)(11.0)} = 1.525\,\text{m/s}$$

The distance once around the carousel is $2\pi r = 2\pi 11.0 = 29.12\,\text{m}$. The time to go once around the carousel is

$$\text{time} = \frac{69.12}{1.525} = \boxed{45.3\,\text{s}}$$

Example (page 145: 40) A motorcycle is traveling up one side of a hill and down the other side. The crest is a circular arc with a radius of 45.0 m. Determine the maximum speed that the cycle can have while moving over the crest without losing contact with the road.

The centripetal force is supplied by gravity. The maximum speed occurs when the force of gravity is equal to the centripetal force.

$$mg = \frac{mv^2}{r}$$

$$v = \sqrt{gr} = \sqrt{(9.80)(45.0)} = \boxed{21.0\,\text{m/s}}$$

Example (page 145: 41) A stone is tied to a string (length = 1.10 m) and whirled in a circle at the same constant speed in two different ways. First, the circle is horizontal and the string is nearly parallel to the ground. Next, the circle is vertical. In the vertical case the maximum tension in the string is 15.0% larger than the tension that exists when the circle is horizontal. Determine the speed of the stone.

Ignore the vertical component of the tension when the stone is moving horizonatally (maybe the stone is sliding over a frictionless table). When in horizontal motion, the tension in the string equals the centripetal force.

$$T = \frac{mv^2}{r}$$

When the stone is in vertical motion, the maximum tension equals the centripetal force plus the force of gravity.

$$T^{\text{MAX}} = \frac{mv^2}{r} + mg$$

Since the tension in vertical motion is 15% larger than the tension in horizontal motion, we have

$$\frac{mv^2}{r} + mg = 1.15\frac{mv^2}{r}$$

After solving this equation we have

$$v = \sqrt{\frac{gr}{0.15}} = \sqrt{\frac{(9.80)(1.10)}{0.15}} = \boxed{8.48\,\text{m/s}}$$

Conceptual Questions

Example The speedometer of your car shows that you are traveling at a constant speed of 35 m/s. Is it possible that your car is accelerating? If so, explain how this could happen.

If I am turning at constant speed, then there is a non-zero acceleration.

Example Consider two people, one on the earth's surface at the equator and the other at the north pole. Which has the larger centripetal acceleration? Explain.

Since centripetal acceleration is mv^2/r and $v = 2\pi r/T$, we have $a = 4\pi^2 r/T^2$. The person at the pole has zero acceleration. The person at the equator has greater acceleration.

Example Other things being equal, would it be easier to drive at high speed around an unbanked horizontal curve on the moon than to drive around the same curve on the earth? Explain.

Friction must supply the centripetal force. Since the normal force on Earth is greater, the maximum frictional force will also be greater. Therefore, it is easier to drive the curve on Earth.

Example A bug lands on a windshield wiper. Explain why the bug is more likely to be dislodged when the wipers are turned on at the high rather than the low setting.

A greater centripetal force is required when the wiper blade is moving at high speed. Since the centripetal force is supplied by friction, there may not be enough friction to keep the bug moving on the circular path.

Example What is the chance of a light car safely rounding an unbanked curve on an icy road as compared to that of a heavy car: worse, the same, or better? Assume that both cars have the same speed and are equipped with identical tires. Account for your answer.

On the unbanked road, the centripetal force is supplied by friction. Thus,

$$\frac{mv^2}{r} = \mu_s mg$$

Since mass cancels from this equation, we see that both cars have the same maximum speed on this curve.

Example A container is filled with water and attached to a rope. Holding the free end of the rope, you whirl the container around in a horizontal circle at a constant speed. There is a hole in the container, so that as you whirl it, water continually leaks out. Explain what, if anything, you feel as the water leaks out.

My arm is supplying the centripetal force which is given by

$$F_c = \frac{mv^2}{r}$$

Since mass is decreasing, I feel a smaller force as water leaks from the bucket.