

Wien's and Stephan-Boltzmann's Laws and the Luminosity Equation

I. Wien's Law (relating color with temperature)

$$\lambda_{\max} = \frac{3 \times 10^6}{T}$$

- Where λ is wavelength in nanometers (nm, billionths of a meter)
- T is temperature, in Kelvins.
- 3×10^6 is a round-off of the proportionality constant which makes the proportional, inverse relationship between λ and T an exact equation.

1 Sample Wien's Law problem for you to try.

1) (1 pt.) What is λ_{\max} for our Sun? (T=5850 K)

Calculate λ_{\max} for a brown dwarf whose T = 2000 K. This is obtained by simply dividing 3×10^6 by the temperature.

$$\lambda_{\max} = \frac{3 \times 10^6}{2000}$$

This, you should be able to solve in your head. A helpful tip: express the temperature in scientific notation.

$$\lambda_{\max} = \frac{3 \times 10^6}{2 \times 10^3} = 1.5 \times 10^3 \text{ or } 1,500 \text{ nm.}$$

2) Suppose a star has three times the temperature of our sun.

a) (1 pt.) What is the wavelength of its λ_{\max} ?

b) (1 pt.) What is the ratio of the star's λ_{\max} compared to our Sun's?

II. Stephan-Boltzmann Law (relating intensity to temperature)

$$I \propto T^4 \text{ or } I = \sigma T^4$$

- Where I is the intensity of light in ergs/s/cm^2
- σ is the constant of proportionality that makes this (I, T) proportional relation an exact equation.
- T is temperature, in Kelvins.

2 Sample problem for S-B Law

1) (1 pt.) How much more intense than the sun's light is the light coming off Sirius's surface? ($T_{\text{Sirius}} = 9900 \text{ K}$)

2) (1 pt.) Our nearest star is the little red M dwarf Proxima Centauri, also known as α Centauri C. Its temperature is only $\sim 2800 \text{ K}$. How many times more intense is sunlight than this star's light, coming off the surface?

3) (1½ pts.) And while we are considering this special star, use the Wien's Law equation to calculate the wavelength of its peak intensity. Is this in the red or infrared part of the electromagnetic spectrum?

I simplify the S-B problems by making them comparative. That is, they ask for the Intensity of some star to be expressed relative to the Sun. (In the case of problem 2, the sun is compared to a fainter star.)

$I \propto T^4$ is the essence of the S-B relation. I varies with T^4 . So, how many times more intense is starlight coming off a hot star of surface temperature 25,000 K than sunlight?

Let I_* represent the intensity of the star and I_{\odot} represent the intensity of sunlight. The problem asks for the intensity of the star expressed relative to the intensity of sunlight. This means a ratio, $I_* : I_{\odot}$ or I_* / I_{\odot} . You get this by subbing in the equivalent values expressed in the right hand side of the S-B equation, σT^4 , expressed also as a ratio:

$$I_* / I_{\odot} = \sigma T_*^4 / \sigma T_{\odot}^4 \quad \sigma \text{ cancels out, so}$$

$$I_* / I_{\odot} = T_*^4 / T_{\odot}^4$$

We could substitute in the temperature values here, but we can simplify the calculation one step further. Recall that the order of multiplication and division is optional.

Expressed as it is above, you must multiply a number by itself 4 times. Get another number and multiply itself 4 times. Then divide the one result by the other. But if you just take the ratio (divide the star's temperature by the sun's temperature first, then you only need to raise the ratio to the 4th power (multiply the ratio by itself 4 times). So,

$$I_* / I_{\odot} = (T_* / T_{\odot})^4$$

Now plug in the temperature values:

$$I_* / I_{\odot} = (25000 / 5850)^4 = 334 : 1$$

III. Luminosity Equation

Every problem here is comparative (L^*/L_\odot), that is, in solar luminosities. This means the constants in the equation cancel out, leaving only the variables, or

$$L \propto AT^4 \text{ or } L \propto R^2T^4$$

- Where L is the luminosity
- A is the surface area ($= 4\pi R^2$)
- R is the radius of the star

The relation or proportional symbol, \propto , means we are only concerned about the properties which can vary with circumstance. Some examples are given below. In all, you are asked to compare luminosities of several major types of stars with the sun. You are solving for luminosity ratios:

$$\frac{L_*}{L_\odot} = \frac{R_*^2 T_*^4}{R_\odot^2 T_\odot^4}$$

1) (1½ pts.) Most Stars are of spectral type M0 V or so. They are about 40% of the size (in terms of radius or diameter) of our sun. Their temperatures are around 3000 K. How intrinsically luminous are they compared to our sun?

3 Sample problem for the Luminosity Equation

Here, I developed already the luminosity equation to get it into the comparative form you need, so I will just give you values for R and T for each star and you can plug them in and solve. As with the three problems you are to solve, I will give you the size ratio but the individual temperatures that you use to determine the temperature ratio.

As before, $T_\odot = 5850 \text{ K}$, $R_\odot = 1$.

I want to acquaint you with the second brightest star in the night sky. It is so far south, we can only barely see it due south in the early evenings of Feb-March. Its apparent magnitude is -0.63.

Canopus is an evolved star. It is an F0 star above the main sequence. Its temperature is 7400 K and it is 64 times larger than the sun. How many times more luminous than the sun is it?

$$\begin{aligned} \frac{L_*}{L_\odot} &= 64^2 \times (7400/5850)^4 = \\ &= 4096 \times 2.56 \\ &= 10,487.3 \text{ or } 10,500\text{x more luminous} \end{aligned}$$

2) (1½ pts.) One class of stars is called red giant. They are about 25x larger than our sun and their temperatures are range around 3000 K. Given these numbers, what is their typical luminosity in solar luminosities?

3) (1½ pts.) A radically different kind of star is the white dwarf. It is typically 1/50 the sun's radius or diameter, but hotter over a range of temperatures Use 15,000 K to calculate how intrinsically luminous are these stars in solar luminosities.

+ 4 points for miscellaneous errors and credit for trying → 15 points total.