Problems Guide **AST 102 Stars, Galaxies, Universe**

for classes of Dr. Gary Mechler
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Though the level of math used in this course is only first-year algebra, and the assigned problems solvable in only one or two steps, a sizable portion of students have trouble figuring their way through the problems and don't even try them or, if they do, can write nothing down. I don't want you to lose a learning opportunity here. One of the Greek contributions to science and learning in general was mathematics. It is essential to your success and empowering. The modern technologies upon which we all depend would be impossible without mathematics. By succeeding in doing these problems, you will be more aware of how we can make specific predictions, and that is the essential practical use of scientific understanding—to predict.

Not all problems assigned involve calculation, but I will still give you some helpful hints for them, for example, the very first problems assigned at the end of Chapter 2.

Assignment 1 No problems assigned.

Assignment 2 No problems assigned.

Assignment 3

Chapter 2, Problems 2, 3, 5: I've given you in the Assignment Sheet the main hint you need to answer these problems—thanks to Table 2.1, there's actually very little calculation here. In lab, students will go forward to learn how to calculate Intensity ratio values, given magnitude difference, but for this assignment, you needn't calculate. Go ahead if you want to. Thought is encouraged in this course.

Chapter 6, Problem 2: Frequency is arrived at by understanding the derivation of the simple equation early in section 6.1. c is the speed of light. Speed is distance per time or distance ÷ time. Taking into account light wave nature, the wavelength provides the useful distance and the frequency the inverse time (a.k.a. "one over time"). That may sound weird, but we use the concept daily in speed concerns. (Frequency is the number of waves passing a particular point per second of time. See how the straightforward multiplication of λ (distance), and frequency, f, give you distance covered per time, c. So, $c = \lambda f$. Solve for λ by dividing both sides of the equation by f. Substitute in the number of waves per second (frequency) given in the problem. The speed of light–do you have it memorized yet? Ok, it's 3×10^8 m/s. So in what unit is your answer, then?

Chapter 7, Problems 1-3: These involve Wien's Law. This is the physics understanding that connected the color (what we see–subjective) of a thermally hot object, like stars, to how hot the object is (an intrinsic property–objective), as measured by surface temperature (Kelvins preferred). The essence of the relation between the two, with color indicated by the wavelength of the peak intensity, is simple: $\lambda_{\max} \propto \frac{1}{T}$. With the constant of proportionality, given its approximate value of 3 million (3×10^6) in the textbook, this relation becomes an exact equation, $\lambda_{\max} = \frac{3{,}000{,}000}{T}$ and you can solve for a specific result, given a specific input value. The proportionality constant assumes that temperature is given in Kelvins and $\lambda_{\max}$ is expressed in nanometers (nm). Problems 1 and 2 have you plug in temperature to get $\lambda_{\max}$, and Problem 3 has you solve for temperature.

Problem 4 uses the Stephan-Boltzmann relation that helps us to make the connection between the temperature of a thermally hot object and the resulting intensity output (energy per unit area), $I \propto T^4$. You don't need the proportionality constant for this problem because it is a comparative one (you just need ratios–constants cancel out when taking ratios), and the relation works as an equation in comparative or changing situations ("before and after"). If the temperature factor is 2, then the relation tells you the intensity varies with T^4; just plug in the 2 and solve for I.

Assignment 3N

Chapter 7, Problem 6: This problem deals with the relation between energy and its corresponding wavelength. That relation is linear and inverse, i.e., $E \propto \frac{1}{\lambda}$ or $\lambda \propto \frac{1}{E}$. In our problem here, the energy is doubled. The relation above tells you what that does to wavelength. Now apply that factor to 500 nm to get your answer.

Problems 7, 8: These are straightforward. All I can add here is that Figure 7.8 is on page 148. For a more visual feel for spectra, look for the spectrum images in Fig. 7-9 on page 150. For problem 8, use also Table 7-1 on page 149.

Problem 9: The author explains this type of solution in the subsection "Calculating the Doppler Velecity" on p. 140. Just remember that $\Delta\lambda$ is $(\lambda - \lambda")$, where $\lambda"$ is the original, or rest, wavelength determined in the lab where the light emitting element is not moving with respect to the observing instrument (spectrograph). A positive result means the observed wavelength is longer (toward the red in the visible part of the em spectrum) than the rest wavelength and the light source is moving away from the observer and vice versa.

Assignment 4

Chapter 1, Problem 7: The text gives you the speed of light in kilometers per second. You could alternatively express that information as the distance traveled in a second and call that distance a light-second. A light-year is the distance traveled by light in a year. So what you need to know is how many seconds are in a year (assume 365¼ days/year) to multiply the distance covered in one second. I'll help you start on this. How many seconds in a minute? Then, how many minutes in an hour? Keep going.

Problem 8: Here's where you see how the light-year unit does double duty. Just keep the definition of light-year in mind (how far light yudda, yudda) and look in the chapter for the rough diameter of our galaxy expressed in light years.

Chapter 9, Problem 1: The relation between parallax and distance is simply an inverse one. No exponents; not even any constant of proportionality when we use the units of arc-seconds and parsecs. So, given the value for distance in pc, just calculate its inverse (reciprocal) and you get its corresponding parallax in the arc-second unit. And vice-versa, when given the parallax value.

For the conversions, remember there are 3.26 ly/parsec times the number of parsecs gives you the number of light years; see how the parsec units cancel, leaving you the ly unit. To convert to astronomical units (AU), remember that in one parsec there are 206,265 AU. You do the same thing as you did in the conversion to ly: 206,265 AU/pc times the number of parsecs. Again, you see how the pc unit neatly cancels out, leaving you the unit you want. Note how the numbers increase as you convert to shorter distance units.

Problem 3: You just deal with the distance and parallax columns here. Since the distances are given in parsecs, the relation between distance and parallax is a simple inverse (or reciprocal, if you prefer the term). To get the parallax equivalent of a given distance, just invert the distance and vice-versa.

Problem 5: The answer comes right out of the definition of absolute magnitude. The only math here is in the conversion from pc to ly, done as in Problem 1.

Problems 6,7: Lab students are welcome to calculate the solar-luminosity values from $2.512^{\Delta M}$, but note these problems are identical with the apparent magnitude problems in Chapter 2. So again, you can just use the Table 2.1 on page 17 to find the solar luminosities corresponding to the magnitude differences.

Problem 8: Spectroscopic parallax is the first calibrated distance determination method I have presented to you. Because it is calibrated, it gets you the distance from the distance modulus equation by giving you the absolute magnitude for a star of a particular temperature/luminosity class. In this case that class is O8 V. Determine its absolute magnitude from the H-R Diagram in Fig. 9.10 on p. 197. Note which luminosity measure to use. To get the distance (in pc), use the correspondence Table 9.1, "Distance Moduli" in Section 9.2, "Intrinsic Brightness".

Problem 10: No hints; you just gotta understand apparent and absolute magnitude.

Problem 15: You'll find the simple equation expressing the relation of luminosity as a function of mass for main sequence stars in the Section 9.5, "A Survey of the Stars". Just substitute the values for *M* and use your $[x^y]$ button or *x^y* sequence, depending upon your calculator.

Assignment 5

Chapter 9, Problem 11: Kepler's Third ("Harmonic") Law, $P^2 = a^3$, with the Period (P) and semi-major axis (a) is a statement originally derived to mathematically describe the orbital speeds of the planets as a function of the size of their orbit. Their motions, we later realized, were due to the sun's gravity. Gravity is directly a property of the mass of the sun. If you follow this, then you realize how Kepler's Third Law is universally applicable to anywhere two masses are close enough to exert a noticeable influence on one another. In double stars, the total mass, therefore, will determine their orbital properties. See if this makes sense to you—with more mass in a system, the objects will need more momentum to keep from falling in. The faster their speeds, the shorter the period of their orbiting. Therefore, we expect to see an inverse relation between mass and orbital period. OK? Now look on page 201 to look at the equation you need here and what do you see? With all this thinking stuff behind us, just a little example of being able to actually figure out the way things work, the solution for the total mass is simple. Though there are exponents involved, they are integers. If you don't have a scientific calculator, just multiply the three *a* values, then divide by the two *P* values. The answer will be not be in fundamental (metric) units, but rather, relative to the sun (solar masses, $M_{\odot}$).

Chapter 10, Problem 2: Gee, another application for Wien's Law, remember, with wavelength in nm and temperature in Kelvins:

$$\lambda_{max} = \frac{3,000,000}{T}$$

Chapter 11, Problem 11: This problem refers to that brightest of the four Trapezium stars, the O6 star, that is the main reason the Orion Nebula shines. Its UV radiation is short enough, due to its temperature (Wien's Law again), and copious enough (due to its tremendous size and temperature) that the hydrogen and other elements surrounding the Trapezium area are excited and ionized. So, at what wavelength does this star peak in its output? This should be getting easy for you now—it's another Wien's Law problem.

Assignment 6

Chapter 12, Problem 2: Make sure you understand (I say that a lot.) the derivation of the relation between mass and lifetime for main-sequence stars on page 247. The resulting relation boils down to an exact equation when we use mass and time units that are based upon the sun's values: $T = \frac{1}{M^{2.5}}$. Note that this calculates only the main-sequence lifetime because this is the only time the star puts out energy at a constant rate (well, pretty constant). Nevertheless, this comprises about 92% of the total lifetime of the star, so it is a reasonably representative value to know. This problem only asks for the main-sequence lifetime.

Just plug in the two given star masses and solve. You notice that the non-integral exponent means you will be helpful–but not actually necessary–to use a scientific calculator for this (the x^y button). If your calculator lacks scientific calculation capability, solve the $M^{2.5}$ part with your knowledge that $M^{2.5} = M^{(2.0+0.5)} = (M^2)(M^{0.5}) = M \times M \times \sqrt{M}$. One final caveat: The final answer should be expressed in years. Take your results in solar lifetimes and multiply by the number of years per solar lifetime. For that conversion, just use 10 billion, 10^{10} years/solar main-sequence liefetime.

Assignment 6N

No hints for these because they; are all extra credit. (Well, there are a couple on the assignment sheet.) If you've been doing the problems so far; go ahead and give these a try. They're not necessarily harder, so just do them. Don't neglect #7 of Chapter 14; it's rather hidden behind a hint on the Assignment Sheet. **IMPORTANT:** I will hand out in class their solutions along with answers to all the questions on the due date, so I must have your work as soon as you arrive in class on that day.

Assignment 7

Chapter 26, Problem 5: The author gives you the needed formula for the volume of a sphere, so just substitute in the 100 ly for *r* and solve to get the volume. Then what? Well, you've now got how many cubic light years there are in the volume of space within 100 ly of our solar system (in ly^3). The author also gives you the average number-density of stars in space like our sun ($\#/ly^3$). If you don't yet see what to do with these two numbers, calculate, using the units to see whether multiplying or dividing them results in simply the # of sun-like stars within 100 ly. Round off your result to two significant figures, because the star density value is only given to that degree of accuracy. (For example, a result of 48.39543 rounds to 48; a result of 395.94393 rounds to 400; and a result of 5938.29838 rounds to 5900.)

Assignment 8

Chapter 15, Problem 2: Because the galaxy is a very thin disk, it is OK to approximate the relative volume we see in it by calculate the relative area of two circles. Just a little simplification, that's all. The area of a circle is πr^2, where r is radius, of course. We see outward from the center of our circle of visibility. Its radius is given as 5 kpc. You find that the diameter of our galaxy is given in the chapter, for example, in the Summary section. (But remember, we're using the radius in the area equation.) Just calculate the areas and compare them: Area of what we see over ("relative to", "divided by") the Area of the galaxy as a whole.

Problem 9: The key here is the # of years it takes to complete one orbit around the galaxy's center, then calculate how many times that value goes into 4.6 billion years. I don't know why the author used such a roundoff value of 5 billion. It implies we don't have its age determined well enough to distinguish between the two possibilities, which is most certainly not true. In any case, this time per orbit is called the *galactic year*. Find its value in the chapter.

Chapter 16, Problem 6: The author gives the hint. Hubble's Law is $V_r = Hd$, where V_r is the measured radial velocity in km/sec. That's given in the problem. *H* is the Hubble constant, given as being 70 km/sec/Mpc, and *d*, well, that's the distance we need to solve for here. Solving boils down to isolating the distance in the Hubble equation above. Since distance is multiplied by *H*, what do you do to *H* to cancel it out? Perform the inverse operation of multiplication on it, then do the same for the left and side; both sides of the equation must maintain their equality. Now plug in and solve for *d*. Your answer is in Mpc–see how that unit results when you rearrange the equation right?

Assignment 8N

Chapter 17, Problem 1: Solve Einstein's mass-energy equivalency equation for mass. Don't worry about the zero subscript–it just means we are dealing with normal inertial mass, not relativistic. Solving for mass (I hope I don't need to say how.) results in some huge figure in kg. You must express *c* in meters, that is, 3×10^8 m/sec to go along with the expression of mass in kg. To convert to mass expressed in solar mass units, you calculate how many times 2×10^{30} kg goes (divides) into that huge result.